TEMPERATURE FIELDS IN STEEL PLATES DURING ACCELERATED INDUCTION HEATING

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The author examines solutions of the problem of nonsteady heat conduction in ferromagnetic plates with internal heat sources, whose intensity depends on the coordinates and time.

So-called accelerated induction heating [1] can be applied in a number of processes requiring thorough heating of metal parts. One such process is the tempering of steel plates. Since the length and width of the plate usually exceed the thickness by more than ten times and the current level in induction heating is constant over the whole surface of the plate, except at the corners, the temperature field in the central part of the plate may be considered one-dimensional and the same as the temperature field for an infinite plate, for which the Fourier heat conduction equation has the form

$$\frac{\partial T}{\partial t} - a\frac{\partial^2 T}{\partial x^2} = \frac{a}{\lambda} W(x, t). \tag{1}$$

Since for all types of steel the tempering temperature is lower than the magnetic transformation temperature, throughout the tempering process the steel preserves its ferromagnetic properties. In the induction heating of a ferromagnetic steel plate the induced current density decreases linearly from the surface of the plate to a layer of metal lying at depth x_1; beyond the limits of a layer of thickness x_1 the induced current density is zero [2]. With such an induced current density distribution the distribution function of the internal heat sources over the thickness of the plate

$$W(x, t) = \begin{cases} \dfrac{3p_0(t)}{x_1}\left(1 - \dfrac{x}{x_1}\right)^2 & \text{for } 0 \leqslant x \leqslant x_1, \\ 0 & \text{for } x_1 \leqslant x \leqslant d/2. \end{cases} \tag{2}$$

The value of the coordinate $x = 0$ corresponds to the surface of the plate, the value $x = d/2$ to the center of the plate. From the data of [2], $x_1 = 1.46\Delta_e$. The depth of penetration of the electromagnetic field into the metal

$$\Delta_e = (\pi f \mu_e \mu_0 \gamma)^{-1/2}.$$

The electrical conductivity of the steel γ and the relative magnetic permeability at the surface of the plate μ_e do not remain constant during the heating process. The electrical conductivity of the steel decreases with increase in temperature, but during accelerated induction heating μ_e increases because of the decrease in specific power. But since in induction heating the surface layer of the steel part is always in a state of magnetic saturation, the degree of increase in μ_e is not great. The opposing nature of the changes in γ and μ_e allows one to assume that $\sqrt{\gamma\mu_e}$ and therefore the quantities Δ_e and x_1, are constant throughout the process of accelerated heating.

The initial conditions are

$$T(x, 0) = 0, \tag{3}$$

i.e., during heating the temperature is calculated from the initial temperature of the plate which is taken as zero.

The boundary conditions are:

$$\frac{\partial T(d/2, t)}{\partial x} = 0, \quad \frac{\partial T(0, t)}{\partial x} = 0. \tag{4}$$

The first of these conditions follows from the symmetry of the temperature distribution over the thickness of the plate; the second boundary condition indicates absence of heat exchange with the environment. Heat losses from the surface of a plate during induction heating in a lined inductor are usually small (particularly at temperatures below the Curie point); moreover, if the heater operates continuously, the temperature of the lining at the commencement of heating is significantly greater than the temperature of the plate; consequently, there is a certain heat flow from the lining to the plate which, after a certain time, reverses direction. All this indicates that the second of conditions (4) holds approximately true.

The problem is solved by the method of separation of variables. Applying this method, we obtain

$$T(x,t)=\frac{d}{2\lambda}\left[\int_0^{\tau}p_0(\nu)\,d\nu+\right.$$

$$\left.+\frac{12}{\pi^2\alpha^2}\sum_{n=1}^{\infty}\frac{\varphi(n,\alpha)}{n^2}\cos n\pi\beta\int_0^{\tau}p_0(\nu)\exp n^2\pi^2(\nu-\tau)\,d\nu\right], \tag{5}$$

where $\varphi(n, \alpha) = 1 - \sin(n\pi\alpha)/n\pi\alpha$, $\alpha = 2x_1/d$ is a parameter characterizing the degree of development of the surface effect, $\beta = 2x/d$ is a relative coordinate, $\tau = 4at/d^2$ dimensionless time (Fourier number), and ν the variable of integration.

In the first stage of accelerated heating, during which the surface temperature of the plate increases to the required final value, the specific power $p_0(\tau) = p_{00}$ is constant. In this case (5) can easily be integrated even for the first stage of heating

$$T(x,t)=\frac{p_{00}d}{2\lambda}\left\{\tau+\frac{12}{\pi^4\alpha^2}\sum_{n=1}^{\infty}\frac{\varphi(n,\alpha)}{n^4}\cos n\pi\beta\left[1-\exp\left(-n^2\pi^2\tau\right)\right]\right\}. \tag{6}$$

Formula (6) is inconvenient for calculations because of the slow convergence of the series. If this series is made to correspond with the expression in square brackets, in the form of a difference of two series, then the series not containing the multiplier $\exp(-n^2\pi^2\tau)$ can be summed. After the necessary transformations, we obtain

$$T(x,t)=\frac{p_{00}d}{2\lambda}\left[\tau+S(\alpha,\beta,\tau)\right], \tag{7}$$

$$S(\alpha,\beta,\tau)=\frac{1}{3}-\beta+\frac{\beta^2}{2}+\frac{\alpha^2}{20}-\psi(\alpha,\beta)-$$

$$-\frac{12}{\pi^4\alpha^2}\sum_{n=1}^{\infty}\frac{\varphi(n,\alpha)}{n^4}\cos n\pi\beta\exp\left(-n^2\pi^2\tau\right),$$

$$\psi(\alpha,\beta)=\begin{cases}\dfrac{\alpha}{4}\left(1-\beta/\alpha\right)^4 & \text{for}\quad 0\leqslant\beta\leqslant\alpha,\\ 0 & \text{for}\quad \alpha\leqslant\beta\leqslant 1.\end{cases}$$

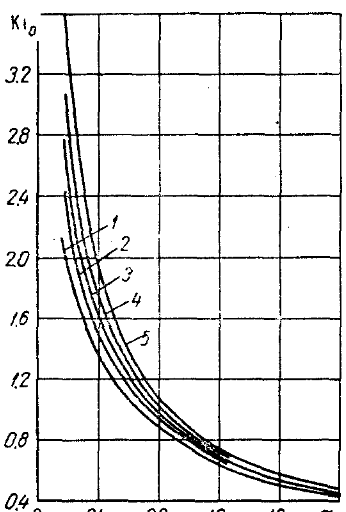

Fig. 1. Dependence of Ki_0 on τ_0: 1) $\alpha = 0$; 2) 0.2; 3) 0.4; 4) 0.6; 5) 1

The series in (7) converges much faster than that in (6). To compute the function $S(\alpha, \beta, \tau)$ when $\tau \geq 0.05$ it is sufficient to take 2-3 terms of the series. To calculate the specific power in the first stage of heating we substitute in (7) the required surface temperature T_0, the relative coordinate of the surface of the plate $\beta = 0$, and the duration of the first stage τ_0. After this substitution we obtain

$$\mathrm{Ki}_0=\left[\tau_0+S(\alpha,0,\tau_0)\right]^{-1}. \tag{8}$$

The dependence of the dimensionless specific power (Kirpichev number) in the first stage of heating $Ki_0 = p_{00}d/2\lambda T_0$ on τ_0 and α is presented in Fig. 1.

In the second stage of accelerated heating it is necessary to regulate the specific power in such a way that the surface temperature of the plate remains constant and equal to T_0, which requires a smooth decrease in specific power. For smooth regulation of the specific power it would be necessary to make an inductor with a conductor of continuously varying width, which involves serious technical difficulties. For accelerated heating Yaitskov [1] used inductors consisting of several discrete sections, connected in series, in each of which the width of the conductor and, consequently, the pitch of the turns and the specific power transmitted to the plate were constant. In the heating process the steel plate travels through the inductor, passing through each section in turn.

If the second stage of heating is separated into k steps of duration $\tau_1, \tau_2, \ldots, \tau_k$, in each of which the specific power is constant, then from (5) it is possible to derive the recursion formula

$$\mathrm{Ki}_i = \left\{1 - \sum_{n=0}^{i-1} \mathrm{Ki}_n \left[\tau_n + S\left(\alpha, 0, \sum_{m=n}^{i} \tau_m\right) - S\left(\alpha, 0, \sum_{m=n+1}^{i} \tau_m\right)\right]\right\} \times$$
$$\times [\tau_i + S(\alpha, 0, \tau_i)]^{-1}, \quad i = 1, 2, \ldots, k, \tag{9}$$

where $\mathrm{Ki}_i = p_{0i} d/2\lambda T_0$, $\tau_i = 4at_i/d^2$ are values of Kirpichev and Fourier numbers for the i-th step in the second heating stage.

By analogous transformations it is possible to find that, for stepwise variation in specific power, the temperature distribution over the thickness of the plate in the i-th step of the second stage of heating

$$T_i(x, t) = T_0 \left\{ \sum_{n=0}^{i-1} \mathrm{Ki}_n \left[\tau_n + S\left(\alpha, \beta, \tau + \sum_{m=n}^{i-1} \tau_m\right) - \right.\right.$$
$$\left.\left. - S\left(\alpha, \beta, \tau + \sum_{m=n+1}^{i-1} \tau_m\right)\right] + \mathrm{Ki}_i \left[\tau + S(\alpha, \beta, \tau)\right]\right\}, \tag{10}$$
$$i = 1, 2, \ldots, k,$$

where $0 \leq \tau \leq \tau_i$.

It is not difficult to see that the product $\mathrm{Ki}_i \tau_i$ is equal to the increase in the mean relative temperature of the plate upon completion of the i-th step. For the entire second stage this increase is

$$\Delta T'_{\mathrm{II}} = \sum_{n=1}^{k} \mathrm{Ki}_n \tau_n = T' - \mathrm{Ki}_0 \tau_0, \tag{11}$$

where $T' = T_m/T_0$ is the required value of the mean relative temperature of the plate at the very end of the heating process.

With stepwise regulation of the specific power the surface temperature of the plate can not remain strictly constant during the second stage of heating. The value of the surface temperature will be equal to T_0 only at the edges of the steps, and inside each step of the second stage a dip in surface temperature will inevitably occur. In order that, for a small number of heating steps, these dips may be small, there must be a smooth change in specific power, i.e., the duration of the steps must increase toward the end of the heating process, and the value of specific power in these steps must decrease correspondingly by the same factor. This requirement will be fulfilled if the increase in the mean temperature of the plate in the second stage of heating is divided equally between all the steps of the second stage, i.e.,

$$\mathrm{Ki}_i \tau_i = \Delta T'_{\mathrm{II}}/k, \quad i = 1, 2, \ldots, k. \tag{12}$$

Combining (9) and (12), we obtain a system of two equations in two unknowns: Ki_i and τ_i. The value of $\Delta T'_{\mathrm{II}}$ is determined from (11). After determining the Kirpichev and Fourier numbers for each of the steps of the second stage from (10), we can find the temperature distribution over the thickness of the plate at any instant of time during the second stage of heating.

The total duration of the process of accelerated heating is

$$\tau_{\Sigma} = \tau_0 + \sum_{i=1}^{k} \tau_i. \tag{13}$$

The more steps there are in the second stage, the more closely will the surface temperature of the plate be maintained constant and, consequently, the shorter the total heating process.

But increasing the number of heating steps increases the complexity of the inductor, which must then consist of a larger number of sections.

For accelerated induction heating of round bars inductors consisting of four sections are employed [1]. In first section the surface temperature is raised, and in the remainder maintained constant. Operation of such inductors shows that, even with three steps in the second heating stage, fluctuations in surface temperature are small and have practically no effect on the duration of heating. Apparently, three steps in the second stage are sufficient for the accelerated heating of steel slabs.

The graphs in Fig. 2 presented the results of solving the system of Eqs. (9) and (12) for each of the three steps in

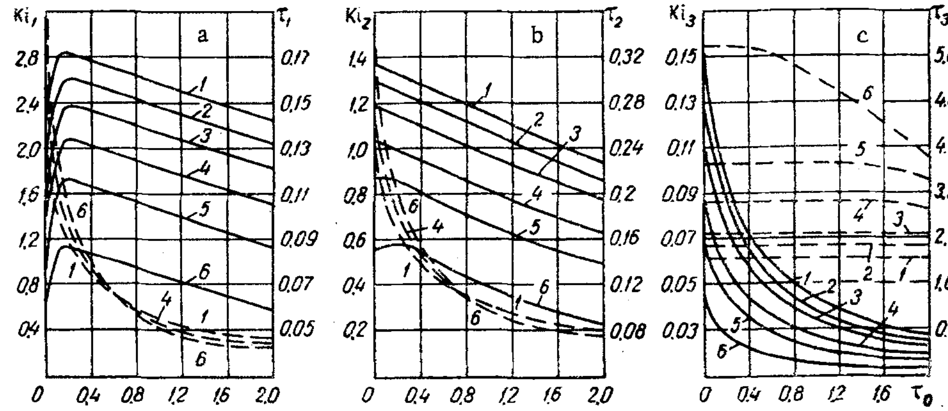

Fig. 2. Dependence of τ_i (continuous lines) and Ki_i (broken lines) on τ_0 for T' = 0.98 in the first (a), second (b), and third (c) steps of the second stage of heating: 1) $\alpha = 0$; 2) 0.1; 3) 0.2; 4) 0.4; 5) 0.6; 6) 1.

the second stage of heating with T' = 0.98. Because the form of the curve of temperature distribution over the thickness of the plate at the very end of the heating process is close to parabolic, at T' = 0.98 the temperature drop between the surface and center of the plate at the end of heating is $\Delta T \approx 0.03T_0$. If, for example, $T_0 \approx 650$°C, then $\Delta T \approx 20$°C. Such a small value of ΔT is perfectly acceptable not only in tempering, but also in any other technological operation.

Using the results of the calculations, we have plotted in Fig. 3 the dependence of the total duration of the accelerated heating process on the duration of the first stage for different α. From Fig. 3 it follows that the minimum duration of accelerated heating is obtained not at $\tau_0 \to 0$, as claimed in [1], but at some finite value of τ_0 which depends on α, i.e., on the degree of development of the surface effect. The minima on the curves in Fig. 3 are attributable to the stepwise regulation of power. If the specific power is continuously regulated, by keeping the surface temperature of the plate in the second stage of heating strictly constant, then the minimum of the total heating time will be at $\tau_0 \to 0$.

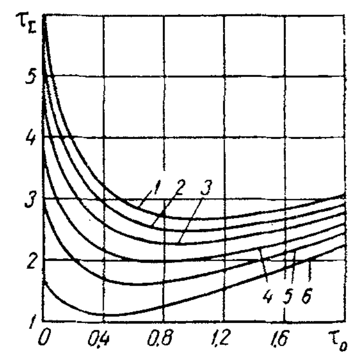

Fig. 3. Dependence of τ_Σ on τ_0 for a three-step second stage of accelerated heating and T' = 0.98: 1-6 see Fig. 2.

Using the graphs of Figs. 1 and 2, we can calculate the accelerated induction heating of steel slabs of any thickness, at any current frequency, without engaging in unwieldy computations based on the formulas presented in this article. The value of τ_0 is determined from Fig. 3 in such a way that the total duration of the heating process is a minimum.

Since the over-all process of accelerated heating was divided into discrete time steps, the relation between which consists only in that the initial condition for any step is the temperature distribution over the thickness of the plate at the end of the previous step, in finding specific values of the power and time from the known values of Ki_i and τ_i, we can use different values of the coefficients α and λ in different steps. Values of these coefficients should be chosen in accordance with the approximately known value of the mean temperature of the plate in each heating step. This substantially increases the accuracy of the calculation, permitting account to be taken of the temperature dependence of the thermophysical properties of steel.

If it is required to calculate accelerated heating for T' ≠ 0.98, it is still possible to use the graphs in Figs. 1, 2a, and 2b, but it is necessary to recalculate values of Ki_3 and τ_3 from the new value of the required increase in the mean temperature of the plate in the final step of the second stage of heating. In this case condition (12) will not be satisfied.

In conclusion, it is necessary to point out that adherence to condition (12) is not mandatory in the calculation of accelerated heating. Condition (12) can be replaced by any other relationship between the quantities Ki_i, τ_i, and $\Delta T'_{II}$. In this case it is necessary only to avoid a sharply irregular distribution of $\Delta T'_{II}$ between the steps of the second stage, since the total duration of the heating process may increase because of the extreme duration of that step in which the greatest fraction of the required increase in mean temperature is imparted to the plate.

NOTATION

T – temperature; t – time; α – thermal conductivity; f – current frequency; μ_0 – $4\pi \cdot 10^{-7}$ henry/m – magnetic permeability of vacuum.

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